OCR
Oxford Cambridge and RSA

Tuesday 11 June 2024 – Afternoon

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \,|\, A) = \mathrm{P}(B)\mathrm{P}(A \,|\, B)$ **or** $\mathrm{P}(A \,|\, B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

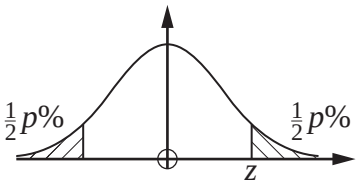
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

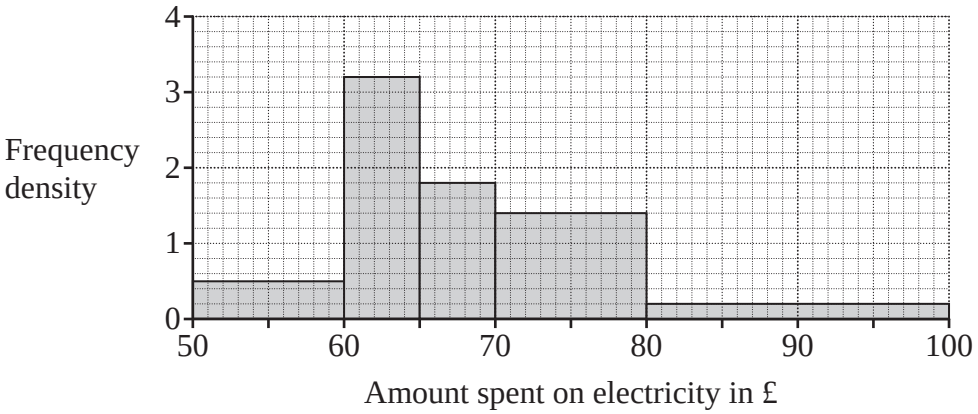
Section A (21 marks)

1 Calculate the exact distance between the points (2, −1) and (6, 1). Give your answer in the form $a\sqrt{b}$, where a and b are prime numbers. **[2]**

2 The equation of a curve is $y = e^x$. The curve is subject to a translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ **and** a stretch scale factor 2 parallel to the y-axis.

Write down the equation of the new curve. **[2]**

3 The histogram shows the amount spent on electricity in pounds in a sample of households in March 2023.



(a) Describe the shape of the distribution. **[1]**

A total of 16 households each spent between £60 and £65 on electricity.

(b) Determine how many households were in the sample altogether. **[2]**

4 **(a)** On the axes in the **Printed Answer Booklet**, sketch the graph of $y = \sin 2\theta$ for $0 \leqslant \theta \leqslant 2\pi$. **[2]**

(b) Solve the equation $\sin 2\theta = -\frac{1}{2}$ for $0 \leqslant \theta \leqslant 2\pi$. **[3]**

5 M is the event that an A-level student selected at random studies mathematics.

C is the event that an A-level student selected at random studies chemistry.

You are given that $P(M) = 0.42$, $P(C) = 0.36$ and $P(M \text{ and } C) = 0.24$. These probabilities are shown in the two-way table below.

	M	M'	Total
C	0.24		0.36
C'			
Total	0.42		1

(a) In the **Printed Answer Booklet**, complete the copy of the two-way table. **[2]**

(b) Calculate the probability that an A-level student selected at random does **not** study chemistry **given that** they do **not** study mathematics. **[2]**

6 The probability distribution of the discrete random variable X is shown in the table.

x	0	1	2	3
$P(X = x)$	0.2	a	$3a$	0.4

(a) Calculate the value of the constant a. **[1]**

(b) A single value of X is chosen at random.

Find the probability that the value is an odd number. **[1]**

(c) **Two** independent values of X are chosen at random.

Calculate the probability that the total of the two values is 3. **[3]**

Section B (79 marks)

7 A sequence is defined by the recurrence relation

$u_{k+1} = u_k + 5$ with $u_1 = -2$.

(a) Write down the values of u_2, u_3, and u_4. **[1]**

(b) Explain whether this sequence is divergent or convergent. **[1]**

(c) Determine the value of u_{30}. **[2]**

(d) Determine the value of $\sum_{k=1}^{30} u_k$. **[2]**

8 The equation of a curve is

$y = 2x^3 + 3mx^2 - 9mx + 4$.

Determine the range of values of m for which the curve has **no** stationary values. **[6]**

9 A teacher is investigating how pupils travel to and from school each day. Pupils can either travel by bus, train, car, bicycle or walk.

The teacher decides to collect a sample of size 60 for the investigation.

(a) The teacher lives in a village 10 miles away from the school.

Explain how collecting a sample which just consists of pupils who live in the same village as the teacher might introduce bias. **[1]**

The table below shows how many students there are in each year.

Year 7	Year 8	Year 9	Year 10	Year 11
86	105	107	101	101

(b) The teacher decides to use the method of proportional stratified sampling.

Calculate the number of pupils in the sample who are in Year 9. **[2]**

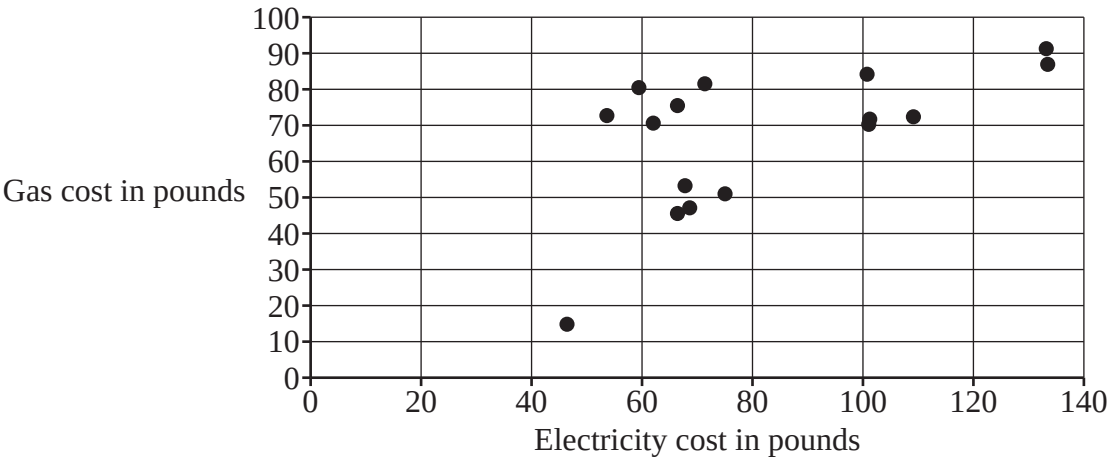
The teacher generates a sample of 10 pupils from the 86 in Year 7 by listing them in alphabetical order and selecting the first name on the list and every ninth name thereafter.

(c) Explain whether this method will generate a simple random sample of the pupils who travel in Year 7. **[1]**

10 **(a)** Determine the first three terms in ascending powers of x of the binomial expansion of $(8+3x)^{\frac{1}{3}}$. **[4]**

(b) State the range of values of x for which this expansion is valid. **[1]**

11 A householder is investigating whether there is any relationship between his monthly cost of gas and his monthly cost of electricity, both measured in pounds (£). The householder collects a random sample of monthly costs and presents them in the scatter diagram below.



One of the points on the diagram represents the energy costs in a month when the householder was away on holiday for three weeks. The other points represent the energy costs in months when the householder did not go away on holiday.

(a) On the copy of the diagram in the **Printed Answer Booklet**, circle the point which represents the month when the householder was most likely to have been away on holiday for three weeks. **[1]**

(b) With reference to the diagram, describe the relationship between the cost of gas and the cost of electricity. **[1]**

The householder decides to test whether there is evidence to suggest that there is any association between the monthly cost of gas and the monthly cost of electricity. The value of Spearman's rank correlation coefficient for this sample is 0.4359 and the associated *p*-value is 0.091 95.

(c) Determine whether there is any evidence to suggest, at the 5% level, that there is any association between the monthly cost of gas and the monthly cost of electricity. **[3]**

12 A survey conducted in 2021 showed that 10% of British adults were vegetarians.

A dietitian believes that the proportion of British adults who are vegetarians may have changed, so decides to conduct a hypothesis test at the 5% level of significance.

In a random sample of 112 adults, the dietitian finds that there are 19 vegetarians.

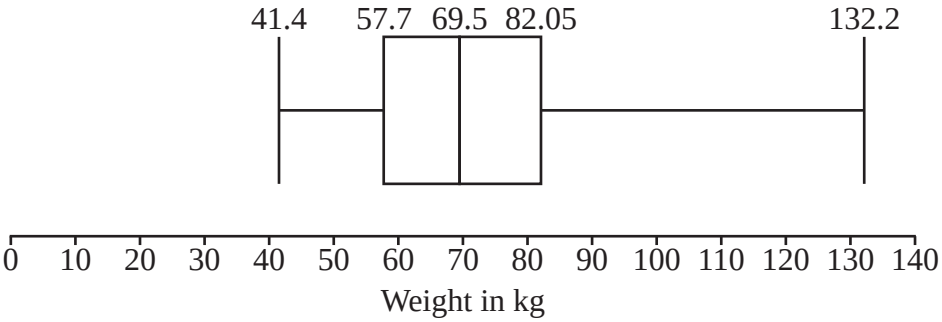
Carry out the hypothesis test to determine whether there is any evidence to support the dietitian's belief. **[7]**

13 Determine the coordinates of the turning points on the curve with equation

$y^2 + xy + x^2 - x = 1.$ **[9]**

14 The pre-release material contains medical data for 103 women and 97 men.

The boxplot represents the weights in kg of 101 of the women from the pre-release material.



(a) Use your knowledge of the pre-release material to give a reason why the weights of all 103 women were **not** included in the diagram. **[1]**

(b) Determine the range of values in which any outliers lie. **[3]**

(c) Use your knowledge of the pre-release material to explain whether these outliers should be removed from any further analysis of the data. **[1]**

(d) The median weight of men in the sample was found to be 79.9 kg.

Explain what may be inferred by comparing the median weight of men with the median weight of women. **[1]**

Further analysis of the weights of both men and women is carried out. The table shows some of the results.

	mean	standard deviation
men	82.69 kg	19.98 kg
women	72.5 kg	19.95 kg

(e) Use the information in the table to make **two** inferences about the distribution of the weights of men compared with the distribution of the weights of women. **[2]**

15 Bottles of Fizzipop nominally contain 330 ml of drink. A consumer affairs researcher collects a random sample of 55 bottles of Fizzipop and records the volume of drink in each bottle.

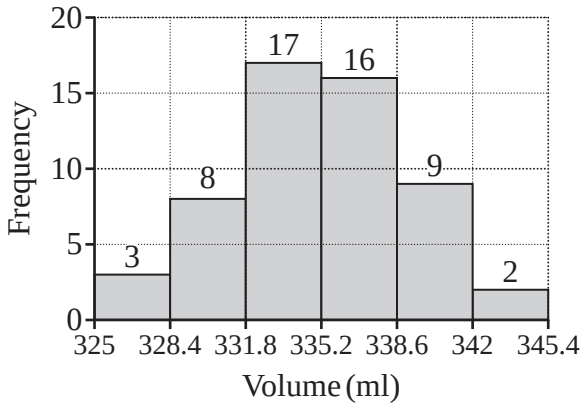
Summary statistics for the researcher's sample are shown in the table.

n	55
$\sum x$	18 535
$\sum x^2$	6 247 066.6

(a) (i) Calculate the mean volume of drink in a bottle of Fizzipop. **[1]**

(ii) Show that the standard deviation of the volume of drink in a bottle of Fizzipop is 3.78 ml. **[1]**

The researcher uses software to produce a histogram with equal class intervals, which is shown below.



(b) Explain why the researcher decides that the Normal distribution is a suitable model for the volume of drink in a bottle of Fizzipop. **[2]**

(c) Use your answers to parts **(a)** and **(b)** to determine the expected number of bottles which contain less than 330 ml in a random sample of 100 bottles. **[3]**

In order to comply with new regulations, no more than 1% of bottles of Fizzipop should contain less than 330 ml.

The manufacturer decides to meet the new regulations by adjusting the manufacturing process so that the mean volume of drink in a bottle of Fizzipop is increased.

The standard deviation is unaltered.

(d) Determine the minimum mean volume of drink in a bottle of Fizzipop which should ensure that the new regulations are met. Give your answer to **3** significant figures. **[3]**

The mean volume of drink in a bottle of Fizzipop is set to 340 ml. After several weeks the quality control manager suspects the mean volume may have reduced. She collects a random sample of 100 bottles of Fizzipop.

The mean volume of drink in a bottle in the sample is found to be 339.37 ml.

(e) Assuming the standard deviation is unaltered, conduct a hypothesis test at the 5% level to determine whether there is any evidence to suggest that the mean volume of drink in a bottle of Fizzipop is less than 340 ml. **[7]**

16 In this question you must show detailed reasoning.

Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{9y}{(x-1)(x+2)},$$

given that $x = 2$ when $y = 16$. **[12]**

END OF QUESTION PAPER